PERFORMANCE OF SHALVI AND WEINSTEIN'S BLIND DECONVOLUTION CRITERIA FOR CHANNELS WITH/WITHOUT ZEROS ON THE UNIT CIRCLE

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ABSTRACT

In this paper, we show that Shalvi and Weinstein's blind deconvolution criteria are applicable for finite SNR regardless of channels having zeros on the unit circle or not. The associated deconvolution filter is stable with a nonlinear relation to the nonblind MMSE equalizer and capable of performing perfect phase equalization for finite SNR.

1. INTRODUCTION

Blind deconvolution (equalization) is a signal processing procedure to restore a source signal $u(n)$ from a given set of measurements

$$x(n) = u(n) * h(n) + w(n) = \sum_k h(k)u(n-k) + w(n) \quad (1)$$

where $h(n)$ is an unknown linear time-invariant (LTI) system (channel) and $w(n)$ is the measurement noise. The blind deconvolution problem occurs in a variety of applications such as communications, seismic exploration, ultrasonic nondestructive evaluation and speech modeling.

Let $v(n)$ be a deconvolution filter and $e(n)$ be the corresponding deconvolved signal, i.e.,

$$e(n) = x(n) * v(n) = u(n) * g(n) + w(n) * v(n) \quad (2)$$

where

$$g(n) = h(n) * v(n) \quad (3)$$

is the overall system after deconvolution. Shalvi and Weinstein [1] find the optimum $v(n)$ by maximizing

$$J_{p,q}(v(n)) = \frac{|C_{p,q}\{e(n)\}|}{[C_{1,1}\{e(n)\}]^{(p+q)/2}}, \quad p+q \geq 3 \quad (4)$$

where $C_{p,q}\{e(n)\}$ denotes the $(p+q)$th-order cumulant of (real or complex) $e(n)$, i.e.,

$$C_{p,q}\{e(n)\} = \text{cum}\{\underbrace{e(n),\ldots,e(n)}_{p \text{ terms}}, \underbrace{e^*(n),\ldots,e^*(n)}_{q \text{ terms}}\} \quad (5)$$

in which the superscript '*' denotes complex conjugation. The criteria $J_{p,q}$ include Wiggins' criterion and Donoho's criteria as special cases [1].

It has been shown in [1] that the optimum $v(n)$ associated with $J_{p,q}$ satisfies the zero forcing (ZF) condition (i.e., $g(n) = \alpha\delta(n-\tau)$), provided that $h(n)$ has no zeros on the unit circle (i.e., the inverse system of $h(n)$ is stable) and that the signal-to-noise ratio

$$\text{SNR} = \frac{E\{|u(n) * h(n)|^2\}}{E\{|w(n)|^2\}} \qquad \text{(see (1))} \quad (6)$$

equals infinity. In practical applications, however, SNR is finite and the behavior of the optimum $v(n)$ associated with $J_{p,q}$ is thus affected by the noise $w(n)$. For the case of real signals, Feng and Chi [2] reported a performance analysis of $J_{p,q}$ for finite SNR when $h(n)$ has no zeros on the unit circle. In this paper, we further extend their analysis to the case of complex signals with $h(n)$ allowed to have zeros on the unit circle. We show that stable $v(n)$ is existent and related to the nonblind minimum mean-square error (MMSE) equalizer [3] in a nonlinear manner along with some properties regarding the behavior of $v(n)$.

2. MODEL ASSUMPTIONS AND REVIEW OF THE MMSE EQUALIZER

For the measurements $x(n)$ given by (1), let us make the following assumptions:

(A1) The channel $h(n)$ is stable with frequency response $H(\omega) = 0$ for $\omega \in \Omega_Z \subset [-\pi, \pi)$, i.e., $\Omega_Z = \{\omega | H(\omega) = 0, \ -\pi \leq \omega < \pi\}$.

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(A2) The source signal $u(n)$ is a zero-mean, independent identically distributed (i.i.d.), non-Gaussian random process with variance $\sigma_u^2 = C_{1,1}\{u(n)\}$.

(A3) The noise $w(n)$ is white Gaussian with variance $\sigma_w^2 = C_{1,1}\{w(n)\}$ and statistically independent of $u(n)$.

The assumption (A1) implies that the inverse system of $h(n)$ is unstable when $\Omega_Z \neq \emptyset$ (an empty set), because $1/H(\omega) = \infty$ for $\omega \in \Omega_Z$. This also means that stable deconvolution filter satisfying the ZF condition does not exist when $\Omega_Z \neq \emptyset$. Next, let us briefly review the MMSE equalizer.

The (infinite-length) MMSE equalizer, which minimizes the mean square error (MSE) $E\{|u(n) - e(n)|^2\}$, is a (noncausal) Wiener deconvolution filter with frequency response given by [4]

$$V_{\mathrm{MSE}}(\omega) = \frac{\sigma_u^2 \cdot H^*(\omega)}{\sigma_u^2 \cdot |H(\omega)|^2 + \sigma_w^2}, \quad \forall \omega \in [-\pi, \pi) \qquad (7)$$

It can be readily seen, from (7), that the MMSE equalizer, $v_{\mathrm{MSE}}(n)$, is a perfect phase equalizer (since $\arg[V_{\mathrm{MSE}}(\omega)] = -\arg[H(\omega)]$) and always stable for finite SNR (since $\sigma_w^2 \neq 0$) regardless of $\Omega_Z = \emptyset$ or $\Omega_Z \neq \emptyset$.

3. ANALYTIC RESULTS

In this section, the behavior of the deconvolution filter $v(n)$ associated with $J_{p,q}$ is analyzed for finite SNR.

A. Behavior of the Deconvolution Filter

Assume that the length of $v(n)$ is doubly infinite so that the analysis of the behavior of $v(n)$ can be performed without taking the effect of finite-length truncation of $v(n)$ into account [1]. With regard to (A1), a property about the existence of stable $v(n)$ associated with $J_{p,q}$ is as follows.

Property 1. When SNR is finite, stable optimum deconvolution filter $v(n)$ associated with $J_{p,q}$ exists with frequency response

$$V(\omega) = 0, \quad \text{for } \omega \in \Omega_Z \qquad (8)$$

□

Moreover, a connection between $v(n)$ and $v_{\mathrm{MSE}}(n)$ is established as follows:

Property 2. The deconvolution filter $v(n)$ associated with $J_{p,q}$ is related to the MMSE equalizer $v_{\mathrm{MSE}}(n)$ via

$$v(n) = A \cdot \{d(n) * v_{\mathrm{MSE}}(n)\} \qquad (9)$$

where A is a real positive constant and

$$d(n) = qB \cdot [g(n)]^p [g^*(n)]^{q-1} + pB^* \cdot [g^*(n)]^{p-1} [g(n)]^q \qquad (10)$$

in which

$$B = \sum_n [g^*(n)]^p [g(n)]^q \qquad (11)$$

□

According to (8), a result about the phase response $\arg[V(\omega)]$ of $V(\omega)$ is as follows:

Property 3. The optimum phase response $\arg[V(\omega)]$ associated with the maximum of $J_{p,q}$ is given by

$$\arg[V(\omega)] = -\arg[H(\omega)] - \omega\xi + \kappa, \quad \text{for } \omega \notin \Omega_Z \qquad (12)$$

where ξ and κ are constants. □

This property implies that the deconvolution filter $V(\omega)$ completely cancels (or equalizes) the channel's phase distortion (up to a time delay ξ and a constant phase shift κ) for $\omega \notin \Omega_Z$ and thus, like the MMSE equalizer, it performs as a perfect phase equalizer.

Let $G_{\mathrm{ZP}}(\omega)$ be a zero-phase system as follows:

$$G_{\mathrm{ZP}}(\omega) = |G(\omega)| = G(\omega) \cdot \exp\{j(\omega\xi - \kappa)\} \qquad (13)$$

(since (12)). Then the impulse response, $g_{\mathrm{ZP}}(n)$, of $G_{\mathrm{ZP}}(\omega)$ possesses the following property:

Property 4. The zero-phase overall system $g_{\mathrm{ZP}}(n)$ associated with $J_{p,q}$ is like an autocorrelation function with $g_{\mathrm{ZP}}(n) = g_{\mathrm{ZP}}^*(-n)$ and

$$g_{\mathrm{ZP}}(0) > |g_{\mathrm{ZP}}(n)|, \quad \forall n \neq 0 \qquad (14)$$

□

This property exhibits the waveshape of $g(n)$ since

$$g(n) = g_{\mathrm{ZP}}(n - \xi) \cdot \exp\{j\kappa\} \quad \text{(by (13))} \qquad (15)$$

Specifically, $|g(n)|$ has a unique maximum at the index $n = \xi$ and meanwhile is symmetric with respect to $n = \xi$.

B. Algorithm for Computing the Theoretical Deconvolution Filter

To verify the proposed analytic results, let us present the following FFT based iterative algorithm for obtaining the theoretical $v(n)$ associated with $J_{p,q}$ from $V_{\mathrm{MSE}}(\omega)$ given by (7) according to Property 2.

Algorithm 1:

(S1) Set $i = 0$. Choose an initial guess $v^{[0]}(n)$ for $v(n)$.

(S2) Set $i = i + 1$. Compute $g^{[i-1]}(n) = h(n) * v^{[i-1]}(n)$. Compute $d(n)$ using (10) and (11) with $g(n) = g^{[i-1]}(n)$ and then compute its L-point DFT $D(\omega_k = 2\pi k/L)$ using FFT.

(S3) Compute $\widetilde{V}(\omega_k) = D(\omega_k) \cdot V_{\rm MSE}(\omega_k)$ (see (9)) and then compute its L-point inverse DFT $\widetilde{v}(n)$ using FFT.

(S4) Compute $v^{[i]}(n) = \widetilde{v}(n)/\sqrt{\sum_n |\widetilde{v}(n)|^2}$. If $\sum_n |v^{[i]}(n) - v^{[i-1]}(n)|^2 > \epsilon$ (a preassigned tolerance for convergence), then go to (S2); otherwise, the theoretical $v(n) = v^{[i]}(n)$ is obtained.

A remark regarding *Algorithm* 1 is as follows:

(R1) *Algorithm* 1 is computationally efficient and never limited by the length of $v(n)$ as long as the FFT length L is chosen sufficiently large so that aliasing effects on the resultant $v(n)$ are negligible.

Next, let us present some simulation results to verify the preceding analytic results.

4. COMPUTER SIMULATION

In the simulation, the source signal $u(n)$ was assumed to be a 4-QAM signal and the channel $H(z) = H_1(z) \cdot H_2(z)$ was taken from [5] where $H_1(z)$ and $H_2(z)$ were causal FIR filters with filter coefficients $\{1, 0, -1\}$ and $\{0.04, -0.05, 0.07, -0.21, -0.5, 0.72, 0.36, 0, 0.21, 0.03, 0.07\}$, respectively. The deconvolution filter $v(n)$ was approximated by a causal FIR filter $\widehat{v}(n)$ of order equal to 30. An iterative gradient-type optimization algorithm with initial condition $\widehat{v}(n) = \delta(n-15)$ was used to find the maximum of $J_{2,2}$ ($p = q = 2$) and the relevant estimate $\widehat{v}(n)$. Then, the average of thirty $\widehat{v}(n)$'s, denoted $\widehat{v}_{\rm ave}(n)$, from thirty independent runs for data length equal to 8000 and SNR = 20 dB (complex white Gaussian noise) was obtained. On the other hand, the theoretical $v(n)$ was obtained using *Algorithm* 1 with $v^{[0]}(n) = \delta(n)$, $L = 1024$ and $\epsilon = 10^{-5}$.

Figures 1(a) and 1(b) depict the magnitude response $|H(\omega)|$ and the principle value, ARG$[H(\omega)]$, of the phase response arg$[H(\omega)]$, respectively, for $\omega \in [-\pi, \pi]$ where a linear phase term in Figure 1(b) was removed for clarity. As indicated in Figures 1(a) and 1(b), $|H(\omega)| = 0$ for $\omega = 0$ and $\pm\pi$ and ARG$[H(\omega)]$ has a discontinuity of π at $\omega = 0$ and a discontinuity of $-\pi$ at $\omega = \pm\pi$ due to the two zeros of $H_1(z)$ on the unit circle ($z = \pm 1$). Figures 2(a), 2(b) and 2(c) show, respectively, the real parts, magnitude responses and phase responses of the obtained $\widehat{v}_{\rm ave}(n)$ (dashed lines) and the theoretical $v(n)$ (solid lines), while their imaginary parts are not displayed because they are almost zero. Note that the scale factor and time delay between $\widehat{v}_{\rm ave}(n)$ and $v(n)$ have been artificially removed. Figure 2(a) reveals that $v(n)$ may be approximated well by a long-length FIR filter $\widehat{v}(n)$ of order equal to about 80, and that the theoretical $v(n)$ obtained by *Algorithm* 1 can serve as a prediction for $\widehat{v}(n)$. Moreover, from Figures 2(b), 2(c) and 1(b), one can see that $|V(\omega = 0)| = |V(\omega = \pm\pi)| = 0$ and ARG$[V(\omega)]$ = $-$ARG$[H(\omega)]$, and that both $|\widehat{V}_{\rm ave}(\omega)|$ and ARG$[\widehat{V}_{\rm ave}(\omega)]$ are close to $|V(\omega)|$ and ARG$[V(\omega)]$, respectively, except for those around $\omega = 0$ and $\pm\pi$. The large magnitude and phase errors around $\omega = 0$ and $\pm\pi$ in Figures 2(b) and 2(c) result from the low magnitude of $H(\omega)$ around these frequencies (see Figure 1(a)) or, equivalently, the low signal power of these frequency components in the data $x(n)$. As a consequence, the results in Figures 2(a), 2(b) and 2(c) are consistent with Properties 1 and 3.

By computing the overall system estimate $\widehat{g}_{\rm ave}(n) = h(n) * \widehat{v}_{\rm ave}(n)$ and the theoretical $g(n)$ using the theoretical $v(n)$, Figure 2(d) shows $|\widehat{g}_{\rm ZP}(n)|$ (dashed line) and $|g_{\rm ZP}(n)|$ (solid line) according to (13). From Figure 2(d), one can see that $|\widehat{g}_{\rm ZP}(n)|$ is quite close to $|g_{\rm ZP}(n)|$ and approaches $\delta(n)$, implying that $\widehat{v}(n)$ performs intersymbol interference (ISI) reduction well for this case. Moreover, $|\widehat{g}_{\rm ZP}(n)|$ is approximately symmetric and $|\widehat{g}_{\rm ZP}(0)| > |\widehat{g}_{\rm ZP}(n)|$, $n \neq 0$, which are consistent with Property 4.

5. CONCLUSIONS

The proposed analytic results about the performance of the blind deconvolution criteria $J_{p,q}$ include the guaranteed stability of the deconvolution filter $v(n)$ regardless of the channel $h(n)$ having zeros on the unit circle or not, the connection of $v(n)$ with the nonblind MMSE equalizer $v_{\rm MSE}(n)$ and the capability of perfect phase equalization for finite SNR, as summarized in Properties 1 through 4. These analytic results are helpful to realizing the behavior of $v(n)$ associated with $J_{p,q}$.

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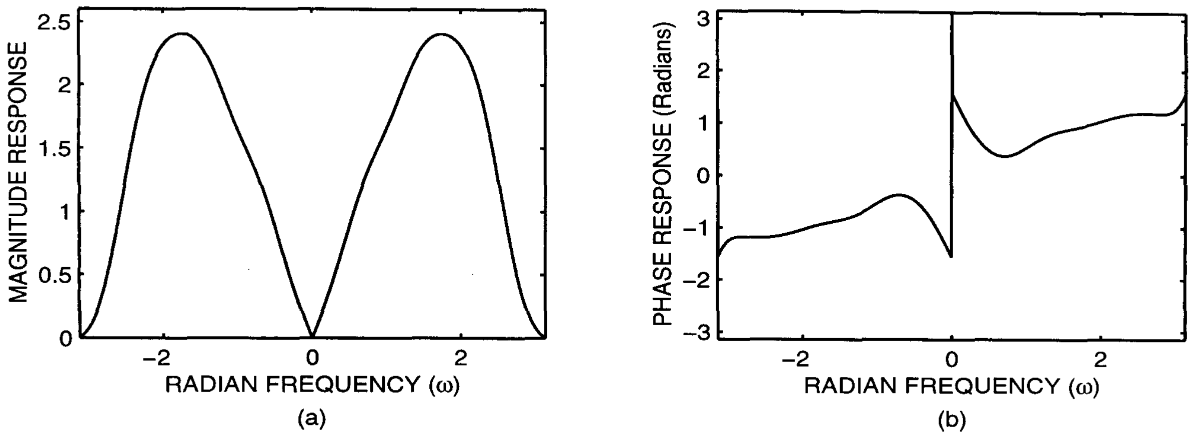

Figure 1. (a) The magnitude response and (b) the phase response of the channel $h(n)$.

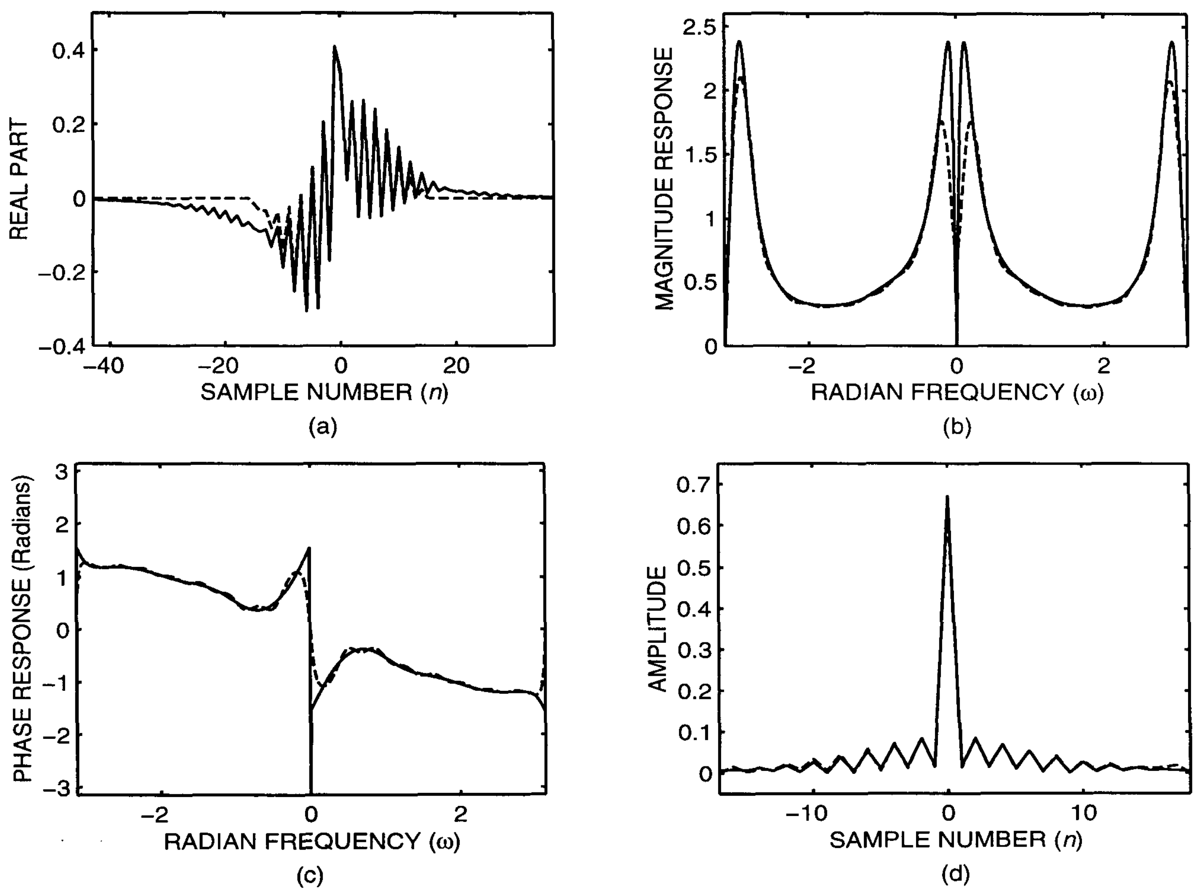

Figure 2. (a) The real parts, (b) the magnitude responses and (c) the phase responses of $\widehat{v}_{\mathrm{ave}}(n)$ (dashed lines) and $v(n)$ (solid lines); (d) $|\widehat{g}_{\mathrm{ZP}}(n)|$ (dashed line) and $|g_{\mathrm{ZP}}(n)|$ (solid line).